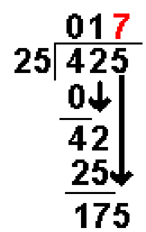
APES MATH TIPS for the AP Exam

APES students are asked to demonstrate their sense of math by calculating their answers by hand and showing work instead of using a calculator. Numbers lose their meaning too often when students become completely calculator-dependent. Practice!

1) **Show all *work*.** No work, no credit.
2) **Show all *units*.** Units provide valuable information.
3) **Be proficient at *unit manipulation***, also called ***dimensional analysis*** or ***factor label***. This is one of the most important math skills, because you will have to fit numbers with units together through multiplication and division to get the desired results.
4) **Know *simple conversion factors*** such as the number of days in a year or hours in a day.
5) Populations to know (approximate): World, U.S., China, India, Indonesia, Brazil
6) ***Add, subtract, multiply,* and *divide* numbers without a calculator.** Multiplication and division are usually seen more than addition and subtraction. The math is able to be done without a calculator, but because students use calculators so much, even advanced students can be awkward when doing long division by hand. Watch the proper placement of the numbers. For 425/25, see the setup from www.mathisfun.com →



    Dividend (long division) is analogous to numerator (fractions)
    Divisor (long division) is analogous to denominator (fractions)

   When dividing by a decimal, move the decimal point to the right in the divisor to create a whole number. Then move the decimal point the same number of places in the dividend.
7) **Develop good *"math sense"* or *"math literacy."*** The answers should make sense. If you calculate a cost of $50 billion per gallon of water, does this seem right?

8) **Know and convert *metric prefixes*.**

T	tera-	10^{12}	(trillion 1,000,000,000,000)
G	giga-	10^{9}	(billion 1,000,000,000)
M	mega-	10^{6}	(million 1,000,000)
k	kilo-	10^{3}	(1000)
h	hecto-	10^{2}	(100)
da	deka-	10^{1}	(10)
d	deci-	10^{-1}	(0.1)
c	centi-	10^{-2}	(0.01)
m	milli-	10^{-3}	(0.001)
µ	micro-	10^{-6}	(one-millionth 0.000001)
n	nano-	10^{-9}	(one-billionth 0.000000001)
p	pico-	10^{-12}	(one-trillionth 0.000000000001)

9) Understand common statistical terms. The **mean** is the mathematical average. The **median** is the 50^{th} percentile, which is the middle value in the distribution of numbers when ranked in increasing order. The **mode** is the number that occurs most frequently in the distribution.

10) **Be comfortable working with *negative numbers*.** Going from -8 ºC to +2 ºC is a 10º change.

11) **Recognize units of *area* and *volume*, and be able to *convert volumes*.**
 1 m = ____ mm… answer → 1000
 $1\ m^3$ = ____ mm^3 answer → $1^3\ m^3 = 1000^3\ mm^3$ $(10^3)^3 = 10^9\ mm^3$
For area conversions, square the number, square the unit. For volume conversions, cube the number, cube the unit.

12) **Calculate *percentages.*** Example: 80/200 = 40/100 = 0.4 = 40%

13) **Work *scientific notation problems* without a calculator.** $M \times 10^n$
Scientific notation does not have to follow the strict format of M being between 1-9.9.
300 million can be written 300×10^6.

a) **Put very large or very small numbers into scientific notation.**
Writing out many zeroes increases the chance of errors.
310,000,000 = 310 million = $310 \times 10^6 = 3.1 \times 10^8$
0.000 000 000 000 097 = 9.7×10^{-14}

b) **Watch decimal places and zeroes in your answer.**
200 x 600 = 120,000 $(2 \times 10^2) \times (6 \times 10^2) = 12 \times 10^4$ which is still 120,000

c) **Multiplication and division** will be common. Multiplying numbers in scientific notation requires the exponents to be added. Dividing numbers in scientific notation requires exponents to be subtracted.

d) **Addition and subtraction** – the exponents must be the same value.
$(3.6 \times 10^5) + (4.9 \times 10^7) = [(0.036 \times 10^2) \times 10^5] + (4.9 \times 10^7) = 4.936 \times 10^7$ OR
$(3.6 \times 10^5) + (4.9 \times 10^7) = (3.6 \times 10^5) + [(490 \times 10^{-2}) \times 10^7 = 493.6 \times 10^5$

14) **Know growth rate calculations.** (see 2003 FRQ #2)
Growth rate = [CRUDE BIRTH RATE + immigration)] – [(CRUDE DEATH RATE + emigration)]
CBR = crude birth rate = # births per 1000, per year
CDR = crude death rate = # deaths per 1000, per year
(CBR – CDR) / 10 = percent change

15) **Calculate percent change:**
a) The rate of change (**percent change**, growth rate) from one period to another =
$[(V_{present} - V_{past}) / V_{past}] * 100$ (where V = value)

b) **Annual rate of change:** take answer from step a) and divide by the number of years between past and present values
Example: A particular city has a population of 800,000 in 1990 and a population of 1,500,000 in 2008. Find the growth rate of the population in this city.

Growth Rate = [(1,500,000 - 800,000) / 800,000] * 100 = 700,000/800,000 * 100 = 87.5% OR

(1,5~~00,000~~ - 8~~00,000~~) x 100 = 15-8 x 100 = 7/8 x 100 = 87.5%
8~~00,000~~ ~~8~~

Average Annual Growth Rate = 87.5% / 18 years = 4.86 %

16) **Calculate percent difference.**

$$\textbf{Percentage Difference} = \left| \frac{\textbf{First Value - Second Value}}{\textbf{(First Value + Second Value) / 2}} \right| \times \textbf{100\%}$$

17) **Know the *Rule of 70* to predict doubling time.**

Doubling time = 70 / annual growth rate (in %, not decimal!) Example: If a population is growing at a rate of 4%, the population will double in 17.5 years. (70 / 4 = 17.5)

18) **Determine *half-life* without a calculator.**

AMOUNT REMAINING = (ORIGINAL AMOUNT)(0.5^x) where x = number of half-lives

Example: A sample of radwaste with a half-life of 10 years has an activity level of 2 Ci (curies). How many years will it take for the sample to have an activity level of 0.25 Ci?

Answer: 2 Ci → 1 Ci (one half-life = 10 yrs.)
1 Ci → 0.5 Ci (another half-life = 10 additional yrs.)
0.5 Ci → 0.25 Ci (another half-life = 10 additional yrs.) = 30 years

19) **Calculate pH using –log [H^+].** $\log_{10} x = y$ and $10^y = x$.

Any pH problems are easily solved without a calculator. Remember that for every one-increment change in pH, the ions change by a factor of 10.
Example: If [H^+] is 10^{-6} M, the pH is 6 and the solution is a weak acid.

20) **Know that "*per capita*" means per person; per unit of population.**

21) **Graphing tips:** include a title and key; set consistent increments for both axes; connect dots for a smooth curve; show dots clearly; know how to use a scatterplot; interpolate and extrapolate; be comfortable with graphing by hand. Almost all APES exam graphs are line graphs.